

Time-Symmetric Approach to Gravity

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Abstract: Quantization of the time symmetric system of interacting strings requires that gravity, just as electromagnetism in Wheeler-Feynman's time symmetric electrodynamics, also be an "adjunct field" instead of an independent entity. The "adjunct gravitational field" emerges, at a scale large compared to that of the strings, as a "statistic" that summarizes how the string positions in the underlying space-time are "compactified" into those in Minkowski space. We are able to show, *without* adding a scalar curvature term to the string action, that the "adjunct gravitational field" satisfies Einstein's equation with no cosmological term.

I. Introduction

In a classic series of papers Wheeler and Feynman showed that their time-symmetric electrodynamicis, where the advanced interaction is equal in strength to the causal retarded interaction, is a viable alternative to the usual theory containing only the retarded interaction [1]. By imposing the complete absorber boundary condition, which requires that there are enough absorbers to absorb all the radiation in the system, they demonstrated that the effect of the advanced electromagnetic interaction on a test charge is completely canceled, except for the radiation reaction on an accelerated one. It is therefore a truly surprising result, although perhaps not generally perceived as such, that Wheeler and Feynman were able to show that the reason Einstein insisted on pursuing his unified field theory in the face of seemingly insurmountable difficulties and widespread skepticism perhaps was his firm belief that matter and field should not be considered as two distinct entities.¹ Since matter is the source of field, neither can exist without the other. He argued therefore that there is only one reality, which happens to have two different aspects; and the theory ought to recognize this from the beginning. He consistently objected to the highly successful quantum field theories, where the electrons and the electromagnetic field, for example, are separately quantized and then coupled together, because he regarded such procedures as improperly doing the same thing twice.

Between field and matter, his choice in favor of the continuous field as the primary concept in spite of the discreteness of quantum phenomena no doubt was influenced by the success of general relativity. He hoped that the quantum discreteness could be derived from generally covariant field equations, which “overdetermine” the fields.² Although the alternative possibility of eliminating the field as an independent entity was actually realized by Wheeler and Feynman in their time-symmetric electrodynamicis,³ Einstein commented that it would be very difficult to make a corresponding theory for the gravitational interaction.⁴

Intuitively it seems easier to construct a continuous field from the discrete matter than the other way around. At a scale large compared to that of the matter constituents, the constructed field will appear to be continuous when the underlying discreteness becomes undetectable. The purpose of this article is to show that it is possible for the string theory generalization of the time-symmetric electrodynamicis of Wheeler and Feynman to construct an “adjunct” gravitational field this way. Because the effect of gravity is so universal and concrete, it may be difficult to imagine gravity not having its own independent degrees of freedom. Nevertheless, as we shall see the elimination of the gravitational field as an independent entity is dictated by the quantization of the time-symmetric system of interacting strings.

We start with our previous suggestion⁵ that the quantization of the time-symmetric system has a statistical origin. In a time-symmetric system, such as Wheeler-Feynman electrodynamicis, the electromagnetic field as already mentioned is not considered an independent entity with its own degrees of freedom. It is instead an “adjunct field” whose present value is a “sufficient statistic” that summarizes all the information about the motion of the charges needed for predicting their future motion.⁶ As a result, the action of a time-symmetric system of interacting strings is exactly additive: the string action is a single sum of partial actions, with each partial action involving only one string and the value of the “adjunct (string) field” evaluated at the position of that string.

The additivity of the string action suggests a connection between the partial action, which is essentially the area of the string world sheet, and entropy.⁵ (A possible connection between area and entropy has been suggested in the different context of black-hole physics.⁷) Based on this connection the classical action has been identified as the total entropy of the system, the action principle of classical mechanics as the equilibrium condition that the total entropy of the system be at a maximum, and the system of interacting strings as being in an equilibrium state of maximum entropy. The same connection also suggests that quantum mechanics is the prescription for calculating the fluctuations of the equilibrium state, with the path-integral representation of the quantum mechanical density matrix element having been shown as an approximation to the partition function of the string theory.⁵

Because the exact additivity of the classical string action is the key to the quantization of a time-symmetric system of interacting strings, gravity in such a system as indicated above must also be an “adjunct field.” Otherwise, the action for the free gravitational field will spoil the exact additivity of the string action. One therefore expects that the “adjunct gravitational field” should again summarize the information about the motion of the strings, and the curvature of space-time is just a reflection of the patterns of the tapestry of motion woven with the world-sheets of the strings.

By studying the free string action, we find that the “adjunct gravitational field” in a time-symmetric system emerges, at a scale much larger than that of the strings, as a “statistic” that summarizes how the string positions in the underlying space-time are “compactified” into those of Minkowski space. At this larger scale, the classical equation of motion for the “adjunct gravitational field” given by the equilibrium condition of maximum entropy is found to be the Einstein equation. It should be emphasized that the Einstein equation is derived here from the kinetic energy term *without* the addition of a scalar curvature term to the string action, in contrast to the usual practice. In fact, the addition of any term corresponding to the action of a free field is strictly forbidden, because it will destroy the additivity of the string action. This restriction leads to the result that there is no cosmological constant term, which together with the fact that the “adjunct fields” have no zero point energy, avoids the difficult problem posed by the stringent upper bound on the cosmological constant.⁸

II. Derivation of Einstein’s Equation

The classical action for a time-symmetric system of free strings propagating in an underlying N-dimensional space-time is given by:⁹

$$S = -T \sum_k \int d^2\sigma_k |det(g_{kk})_{ab}|^{1/2} \quad (1)$$

where $(g_{kk})_{ab} = \eta_{mn} \partial_a \xi_k^m \partial_b \xi_k^n$. In Eq.(1) one sums over all the strings in the universe in order to ensure that the complete absorber condition is satisfied. This condition, which requires that there must be enough absorbers to absorb all the radiation in the system, maintains macroscopic causality of the time-symmetric system, whose interaction term

necessarily contains both retarded and advanced interactions.³ The constant T has the dimension of inverse length squared; $d^2\sigma_k$ is the surface element of the world-sheet of string k , which is described by specifying $\xi_k^m(\sigma_k^0, \sigma_k^1)$, the string position at given values of σ_k^0 and σ_k^1 ; $\partial_a \xi_k^m = \partial \xi_k^m / \partial \sigma_k^a$; the indices $a, b = 0, 1$; η_{mn} is the metric of the underlying space-time; and the indices $m, n = 0, \dots, N-1$.

By the introduction of a metric $(h_{kk})_{ab}$ for the world-sheet of string k , one arrives at an equivalent formula that avoids the awkward square root involving the ξ_k 's:⁹

$$S = -T \sum_k \int d^2\sigma_k h_{kk}^{1/2} h_{kk}^{ab} \eta_{mn} \partial_a \xi_k^m \partial_b \xi_k^n,$$

where h_{kk}^{ab} is the inverse of $(h_{kk})_{ab}$ and h_{kk} is the absolute value of the determinant of $(h_{kk})_{ab}$. At the extremum of the action, the metric $(h_{kk})_{ab}$ is proportional to $(g_{kk})_{ab}$, and the action given above becomes equal to that given by Eq.(1). By going to the conformal gauge, where $(h_{kk})_{ab} = \exp(\phi_k) \eta_{ab}$, η_{ab} is the metric of a flat world-sheet and $\exp(\phi_k)$ is a conformal factor, we obtain a simplified string action independent of $\exp(\phi_k)$:

$$S = -T \sum_k \int d^2\sigma_k \eta^{ab} \eta_{mn} \partial_a \xi_k^m \partial_b \xi_k^n. \quad (2)$$

The free string action is a direct generalization of the kinetic energy term of the particles, with the area of the string world-sheets replacing the length of the world-lines of the particles. Because the idea that an ‘‘adjunct gravitational field’’ can emerge from such a kinetic energy term is perhaps surprising, it may be worthwhile to describe in detail the steps involved.

The first step is the usual idea of ‘‘compactification,’’ whereby the positions of the strings in the underlying space-time are ‘‘compactified’’ into the string positions in Minkowski space, x_k . In other words, the N components of ξ_k only depends on the four components of x_k : $\xi_k^m[x_k^\mu(\sigma_k^a)], \mu = 0, \dots, 3$. If Minkowski space were to have the same number of dimensions as the underlying space-time instead of the observed four dimensions, then compactification would simply be a coordinate transformation in the underlying space-time.

Next, we separate the string position x_k into two parts:

$$x_k(\sigma_k^\mu) = x_{ko}(\tau_k) + y_k(\sigma_k^a),$$

where $x_{ko} = x_k(\sigma_k^0, \sigma_k^1 = 0)$ is a particle-like path depending on only one variable, which will be denoted by the proper time τ_k ; and $y_k = x_k(\sigma_k^a) - x_{ko}$. At a scale much larger than the length of the string, y_k will appear to be a small string jitter that varies rapidly about the particle-like path as σ_k^1 ranges over the entire length of the string.

The string action given by Eq.(2) can be expressed in terms of the string positions, x_k :

$$S = -T \sum_k \int d^2\sigma_k \eta^{ab} (g_{kk})_{\mu\nu} \partial_a x_k^\mu \partial_b x_k^\nu,$$

where $(g_{kk})_{\mu\nu} = \eta_{mn}(\partial\xi_k^m/\partial x_k^\mu)(\partial\xi_k^n/\partial x_k^\nu)$. At a scale much larger than that of the strings, one can treat the particle-like path of the strings as a continuous variable and introduce an “adjunct gravitational field” $g_{\mu\nu}(x_o)$ by interpolating $(g_{kk})_{\mu\nu}|_{y_k=0}$. The four-dimensional space-time metric $g_{\mu\nu}$ will have the required ten algebraically independent components, if the dimension of the underlying space-time is greater than or equal to ten.

Because the number of strings in the universe is necessarily a large number, the same large number of particle-like paths is included in the sum over the strings. It should therefore be a good approximation to replace the sum over the strings by the functional integral summing all particle-like paths, and we drop the string label k . The separation of each string position into two parts, as was done above, allows us to expand the integrand in powers of the rapidly varying string jitters. Using the requirement of general covariance, we adopt the technique used by Polyakov in treating the long-wavelength oscillations of strings and introduce the covariant derivative in the y -direction:¹⁰

$$\nabla_y(\partial_a x_o^\mu) = y^\alpha \nabla_\alpha(\partial_a x_o^\mu) = y^\alpha [\partial(\partial_a x_o^\mu)/\partial x_o^\alpha + \{\frac{\mu}{\alpha\rho}\}(\partial_a x_o^\rho)],$$

where the Christoffel-bracket

$$\{\frac{\rho}{\alpha\beta}\} = (1/2)g^{\rho\sigma}(\partial g_{\sigma\beta}/\partial x_o^\alpha + \partial g_{\alpha\sigma}/\partial x_o^\beta - \partial g_{\alpha\beta}/\partial x_o^\sigma).$$

Because $\{\frac{\rho}{\alpha\beta}\}$ is symmetric in the indices α and β (absence of torsion), we have

$$\nabla_y(\partial_a x_o^\mu) = (\nabla_a y)^\mu = \partial_a y^\mu + \{\frac{\mu}{\alpha\beta}\}(\partial_a x_o^\alpha)y^\beta.$$

The string action expanded to first order of the string jitters is given by:

$$S \approx -T \int D[x_o] \int d^2\sigma \eta^{ab} [g_{\mu\nu}(x_o) \partial_a x_o^\mu \partial_b x_o^\nu + 2g_{\mu\nu}(x_o) \partial_a x_o^\mu (\nabla_b y)^\nu].$$

The classical equations of motion is given by the action principle, which is the equilibrium condition of maximum entropy. Setting $\delta S = 0$, under the infinitesimal variation $x_o + y \rightarrow x_o + y + \epsilon y'$, we obtain at the lowest order of the above expansion:¹⁰

$$\int D[x_o] \int d^2\sigma \eta^{ab} g_{\mu\alpha} \nabla_a(\partial_b x_o^\mu) y'^\alpha = 0,$$

where $\nabla_a(\partial_b x_o^\mu) = \partial_a \partial_b x_o^\mu + \{\frac{\mu}{\alpha\beta}\} \partial_a x_o^\alpha \partial_b x_o^\beta$. The equation of motion, expressed in terms of the proper time, is:

$$d^2 x_o^\mu / d\tau^2 + \{\frac{\mu}{\alpha\beta}\} (dx_o^\alpha / d\tau) (dx_o^\beta / d\tau) = 0. \quad (3)$$

Eq.(3) describes the particle-like paths of the strings as the geodesics of $g_{\mu\nu}$, which justifies the identification of $g_{\mu\nu}$ as the (adjunct) gravitational field.

Using Eq.(3) and the fact that the commutator of two covariant derivatives is the curvature tensor, we find at the next order of the above expansion:¹⁰

$$\int D[x_o] \int d^2\sigma \eta^{ab} [g_{\alpha\beta} (\nabla_a y)^\alpha (\nabla_b y')^\beta + R_{\alpha\mu\beta\nu} y^\alpha y'^\beta \partial_a x_o^\mu \partial_b x_o^\nu] = 0. \quad (4)$$

The first term in the square bracket gives an equation for the string jittery. The second term gives the equation of motion at the large scale that we are looking for. With the average (obtained by integrating over σ^1) of the jitters denoted by: $(1/2) \langle y^\alpha y'^\beta + y^\beta y'^\alpha \rangle = g^{\alpha\beta} \phi$, where ϕ is a scalar, the equation of motion is:

$$R_{\mu\nu} = 0. \quad (5)$$

Eq.(5) is the Einstein vacuum equation, with no cosmological constant term. If we had included the string interaction term, the righthand side of Eq.(4) would not be zero. In that case, equating the second term on the lefthand side of Eq.(4) to the terms on the righthand side which would also be multiplied by the scalar ϕ , we obtain the Einstein equation.

In conclusion, we suggest that gravity just as electromagnetism in a time-symmetric system is an “adjunct field,” not an independent entity. It emerges, at a scale much larger than the length of the strings, as a “statistic” that summarizes how the string positions in the underlying space-time are “compactified” into those in Minkowski space. The “adjunct gravitational field” at the larger scale satisfies Einstein’s equation, with no cosmological term. The classical string action with gravity remains exactly additive. Because of the exact additivity of the string action, quantization of the time-symmetric system in the presence of gravity vcan proceed in the same way proposed previously⁵ without introducing any new difficulty of principle.

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